5) Evaluate the surface integral $\iint_S F.ndA$ for the vector $F=2i+5j+3k$;

where the surface S is the portion of the cone $z^2=x^2+y^2$ that lies inside the cylinder

$x^2+y^2=1$ and the planes z = 0 and z = 1. The parametric representation of a cone is given by: $r(r,\theta)=[r\cos\theta, r\sin\theta, r]$.

Solution:

Parametric representation of a cone:

$$(x,y,z)\longrightarrow \quad (r\cos\theta, r\sin\theta, r)$$

$$N=\vec{r}_r\times\vec{r}_\theta$$

$$\vec{r}_r=(\cos\theta,\sin\theta,1)$$

$$\vec{r}_\theta=(-r\sin\theta, r\cos\theta, 0)$$

$$N=\vec{r}_r\times\vec{r}_\theta$$

$$N=\begin{vmatrix} i & j & k \\ \cos\theta & \sin\theta & 1 \\ r\sin\theta & r\cos\theta & 0 \end{vmatrix}$$

$$=[-r\cos\theta i, r\sin\theta, r]$$

$$F(r(r,\theta))=[2,5,3]$$

$$\text{F.N} = -2r\sin\theta-5\sin\theta+3\text{r}$$

$$\iint_S F.ndA=\iint_R F\left(\vec{r}(r,\theta)\right).N(r,\theta)\,drd\theta$$

$$¿\int_0^1\iint (-2r\cos\theta-5r\sin\theta+3r)\,drd\theta$$

$$¿\int_0^{2\pi}(\frac{-2}{2}r^2cos\theta-\frac{5}{2}r^2+\frac{3}{2}r^2¿)¿$$ Limits for r = 0 and 1

$$¿\int_0^{2\pi}\left(-cos\theta-\frac{5}{2}sin\theta+\frac{3}{2}\right)d\theta$$

$$¿\left(-sin\theta+\frac{5}{2}cos\theta+\frac{3}{2}\theta\right)\ Limits for\ \theta 0¿2\pi$$

$$=\left(0+\frac{5}{2}+3\pi\right)-\left(0+\frac{5}{2}+0\right)$$

$$¿3\pi$$